ON THE CLEANNESS OF CUSPIDAL CHARACTER SHEAVES

G. Lusztig

Abstract. We prove the cleanness of cuspidal character sheaves in arbitrary characteristic in the few cases where it was previously unknown.

1. Statement of results

1.1. Let $\mathbf{k}$ be an algebraically closed field of characteristic exponent $p \ge 1$. Let G be a connected reductive algebraic group over $\mathbf{k}$ with adjoint group G_{ad}. It is known that, if A is a cuspidal character sheaf on G, then $A = IC(\bar{\Sigma}, \mathcal{E})[\dim \Sigma]$ where Σ is the inverse image under $G \to G_{ad}$ of a single conjugacy class in G_{ad}, $\mathcal{E}$ is an irreducible local system on Σ equivariant under the conjugation G-action and IC denotes the intersection cohomology complex. (For any subset γ of G we denote by $\bar{\gamma}$ the closure of γ in G.) We say that A is clean if $A|_{\bar{\Sigma}-\Sigma} = 0$. This paper is concerned with the following result.

Theorem 1.2. *Any cuspidal character sheaf of G is clean.*

By arguments in [L2, IV, §17] it is enough to prove the theorem in the case where G is almost simple, simply connected. In this case the theorem is proved in [L2,V,23.1] under the following assumption on p: if $p = 5$ then G is not of type E_8; if $p = 3$ then G is not of type E_7, E_8, F_4, G_2; if $p = 2$ then G is not of type E_6, E_7, E_8, F_4, G_2. In the case where $p = 5$ and G is of type E_8 or $p = 3$ and G is of type E_7, E_8, F_4, G_2 or $p = 2$ and G is of type E_6, E_7, G_2, there are some cuspidal character sheaves on G for which the arguments of [L2, V, §23] do not apply, but Ostrik [Os] found a simple proof for the cleanness of these cuspidal character sheaves. The proof of the theorem in the remaining case ($p = 2$ and G of type E_8 or F_4) is completed in 3.8; in the rest of this section we place ourselves in this case.

Note that a portion of our proof relies on computer calculations (via the reference to [L4] in 2.4(a) and the references to [L3], [L5]).

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1.3. For any complex of $\bar{\mathbf{Q}}_l$-sheaves K on G let $\mathcal{H}^iK$ be the i-th cohomology sheaf of K and let ${}^pH^iK$ be the i-th perverse cohomology sheaf of K. If M is a perverse sheaf on G and A is a simple perverse sheaf on G let $(A:M)$ be the number of times that A appears in a Jordan-Hölder series of M. We write "G-local system" instead of "G-equivariant $\bar{\mathbf{Q}}_l$-local system for the conjugation action of G". We set $\Delta = \dim G$.

The next two properties are stated for future reference.

(a) *Let A be a cuspidal character sheaf on G and let X be a noncuspidal character sheaf on G. Then $H_c^*(G, A\otimes X) = 0$.*
(See [L2, II, 7.2].)

(b) *Let γ be a unipotent class in G and let $\mathcal{L}$ be an irreducible noncuspidal G-local system on γ. Then there exists a noncuspidal character sheaf X of G such that* $\text{supp}(X)\cap G_u \subset \bar{\gamma}$ *and $X|_\gamma = \mathcal{L}[d]$ for some $d \in \mathbf{Z}$.*
(See [L1, 6.5].)

1.4. From the results already quoted we see that if G' is the centralizer of a semisimple element $\neq 1$ of G, then any cuspidal character sheaf of G' is clean. Using [L2, II, 7.11] we then see that any cuspidal character sheaf of G with non-unipotent support is clean. Thus it is enough to prove the cleanness of cuspidal character sheaves with support contained in G_u, the unipotent variety of G. For any $i \in \mathbf{N}$ we denote by γ_i a distinguished unipotent class in G of codimension i (assuming that such class exists); note that γ_i is unique if it exists. According to Spaltenstein [Sp1, p.336], γ_i carries an irreducible cuspidal local system precisely when $i \in I$ where $I = \{10, 20, 22, 40\}$ (type E_8) and $I = \{4, 6, 8, 12\}$ (type F_4); this cuspidal local system (necessarily of rank 1) is unique (up to isomorphism) and denoted by $\mathcal{E}_i$ except if $i = 10$ (type E_8) and $i = 4$ (type F_4) when there are two non-isomorphic irreducible cuspidal local systems on γ_i denoted by $\mathcal{E}_i, \mathcal{E}_i'$. We can then form the four admissible (see [L2, I, (7.1.10)]) complexes $A_i = IC(\bar{\gamma}_i, \mathcal{E}_i)[\dim \gamma_i]$ ($i \in I$) on G and the admissible complex $A_i' = IC(\bar{\gamma}_i, \mathcal{E}_i')[\dim \gamma_i]$ (where $i = 10$ for type E_8, $i = 4$ for type F_4). According to Shoji [Sh2] these five admissible complexes are character sheaves on G; they are precisely the character sheaves on G with support contained in G_u. From [L2, II, 7.9] we see that A_{40} is clean (type E_8) and A_{12} is clean (type F_4). According to Ostrik [Os], A_{10}, A_{10}', A_{22} are clean (type E_8) and A_4, A_4', A_6 are clean (type F_4). Moreover, from [Os] it follows that

(a) *if G is of type E_8 and $i \in \mathbf{Z}$ then $\mathcal{H}^i(A_{20})|_{\gamma_{22}}$ does not contain $\mathcal{E}_{22}$ as a summand.*

1.5. We show:

(a) *Let $i = 20$ (type E_8), $i = 8$ (type F_4). Let $A = A_i$. Let γ be a unipotent class of G. Then $\oplus_j \mathcal{H}^j A|_\gamma$ does not contain any irreducible noncuspidal G-local system as a direct summand.*

Assume that this is not true and let γ be a unipotent class of minimum dimension such that $\oplus_j \mathcal{H}^j A|_\gamma$ contains an irreducible noncuspidal G-local system, say $\mathcal{L}$, as a direct summand. We can assume that $\mathcal{H}^{i_0} A|_\gamma$ contains $\mathcal{L}$ as a direct summand

and that for $j > i_0$, $\mathcal{H}^j A|_\gamma$ is a direct sum of irreducible cuspidal G-local systems on γ. Clearly,

(b) *for any unipotent class $\gamma' \subset \bar\gamma - \gamma$, $\oplus_j \mathcal{H}^j A|_{\gamma'}$ is a direct sum of irreducible cuspidal G-local systems.*

We can assume that $\gamma \subset \bar\gamma_i$; if $\gamma = \gamma_i$ the result is obvious so that we may assume that $\gamma \subset \bar\gamma_i - \gamma_i$. By 1.3(b) we can find a a noncuspidal character sheaf X of G such that $\text{supp}(X) \cap G_u \subset \bar\gamma$ and $X|_\gamma = \mathcal{L}^*[d]$ for some $d \in \mathbf{Z}$. By 1.3(a) we have $H_c^*(G, A \otimes X) = 0$. Since $\text{supp}(A) \subset G_u$ it follows that $H_c^*(G_u, A \otimes X) = 0$. Since $\text{supp}(X) \cap G_u \subset \bar\gamma$ it follows that $H_c^*(\bar\gamma, A \otimes X) = 0$.

We show that $H_c^*(\bar\gamma - \gamma, A \otimes X) = 0$. It is enough to show that for any unipotent class $\gamma' \subset \bar\gamma - \gamma$ we have $H_c^*(\gamma', A \otimes X) = 0$. Using (b) we see that it is enough to show that for any irreducible cuspidal G-local system $\mathcal{E}'$ on γ' we have $H_c^*(\gamma', \mathcal{E}' \otimes X) = 0$. We can find a cuspidal character sheaf A' on G such that $\text{supp}(A') = \bar\gamma'$, $A'|_{\gamma'} = \mathcal{E}'[\dim \gamma']$. Then A' must be A_{40} or A_{22} (for type E_8) and A_{12} (for type F_4); in particular A' is clean. Hence

$H_c^*(\gamma', \mathcal{E}' \otimes X) = H_c^*(\bar\gamma', A' \otimes X) = H_c^*(G, A' \otimes X)$

and this is 0 by 1.3(a).

From $H_c^*(\bar\gamma, A \otimes X) = 0$, $H_c^*(\bar\gamma - \gamma, A \otimes X) = 0$ we deduce that $H_c^*(\gamma, A \otimes X) = 0$ that is $H_c^*(\gamma, A \otimes \mathcal{L}^*) = 0$. Let $\delta = \dim \gamma$. We have $H_c^{2\delta + i_0}(\gamma, A \otimes \mathcal{L}^*) = 0$. We have a spectral sequence with $E_2^{r,s} = H_c^r(\gamma, \mathcal{H}^s(A) \otimes \mathcal{L}^*)$ which converges to $H_c^{r+s}(\gamma, A \otimes \mathcal{L}^*)$.

We show that $E_2^{r,s} = 0$ if $s > i_0$. It is enough to show that $H_c^*(\gamma, \mathcal{E}'' \otimes \mathcal{L}^*) = 0$ for any irreducible cuspidal G-local system $\mathcal{E}''$ on γ. We can find a cuspidal character sheaf A'' on G such that $\text{supp} A'' = \bar\gamma$, $A''|_\gamma = \mathcal{E}''[\delta]$. Since $\gamma \subset \bar\gamma_i - \gamma_i$ we see that A'' must be A_{40} or A_{22} (type E_8) or A_{12} (type F_4) so that A'' is clean. Hence

$H_c^*(\gamma, \mathcal{E}'' \otimes \mathcal{L}^*) = H_c^*(\bar\gamma, A'' \otimes X) = H_c^*(G, A'' \otimes X)$

and this is 0 by 1.3(a).

We have also $E_2^{r,s} = 0$ if $r > 2\delta$. It follows that

$E_2^{2\delta, i_0} = E_3^{2\delta, i_0} = \dots = E_\infty^{2\delta, i_0}$.

But $E_\infty^{2\delta, i_0}$ is a subquotient of $H^{2\delta + i_0}(\gamma, A \otimes \mathcal{L}^*)$ hence it is zero. It follows that $0 = E_2^{2\delta, i_0} = H_c^{2\delta}(\gamma, \mathcal{H}^{i_0}(A) \otimes \mathcal{L}^*)$. Since $\mathcal{L}$ is a direct summand of $\mathcal{H}^{i_0}(A)$ it follows that $H_c^{2\delta}(\gamma, \mathcal{L} \otimes \mathcal{L}^*) = 0$. This is clearly a contradiction. Thus (a) is proved.

1.6. We show:

(a) *Let A be a cuspidal character sheaf on G such that* $\text{supp}(A) = \bar\gamma$, *$\gamma$ a unipotent class in G; let $\mathcal{E}$ be an irreducible G-local system on γ such that $A|_\gamma = \mathcal{E}[\delta]$, $\delta = \dim \gamma$. Let Y be a noncuspidal character sheaf of G. Then $\oplus_j \mathcal{H}^j Y|_\gamma$ does not contain $\mathcal{E}^*$ as a direct summand.*

Assume that $\oplus_j \mathcal{H}^j Y|_\gamma$ contains $\mathcal{E}$ as a direct summand. We can find i_0 such that $\mathcal{H}^{i_0} Y|_\gamma$ contains $\mathcal{E}$ as a direct summand but $\mathcal{H}^j Y|_\gamma$ does not contain $\mathcal{E}$ as a direct summand if $j > i_0$. We have $H_c^{2\delta}(\gamma, \mathcal{E} \otimes \mathcal{H}^{i_0} Y) \neq 0$. By 1.3(a) we have $H_c^*(G, A \otimes Y) = 0$ hence $H_c^*(\bar\gamma, A \otimes Y) = 0$. We show that

(b) $H_c^*(\bar\gamma-\gamma, A\otimes Y)=0$.

If A is clean then (b) is obvious. Thus to prove (b) we may assume that $A=A_{20}$ (type E_8) and $A=A_8$ (type F_4). It is enough to show that for any unipotent class $\gamma'\subset\bar\gamma-\gamma$ we have $H_c^*(\gamma',A\otimes Y)=0$. It is enough to show that $H_c^*(\gamma',\mathcal{H}^j(A)\otimes Y)=0$ for any j. If $\mathcal{H}^jA|_{\gamma'}=0$, this is obvious. Thus we may assume that $\mathcal{H}^jA|_{\gamma'}\neq 0$. By 1.5(a), $\mathcal{H}^jA|_{\gamma'}$ is a direct sum of (at least one) copies of irreducible cuspidal G-local systems on γ'. It follows that $\gamma'=\gamma_{40}$ (type E_8) and $\gamma'=\gamma_{12}$ (type F_4); we use that in type E_8 we have $\gamma'\neq\gamma_{22}$; see 1.4(a). It is then enough to show that $H_c^*(\gamma',\mathcal{E}'\otimes Y)=0$ where $\mathcal{E}'$ is $\mathcal{E}_{40}$ (type E_8) and $\mathcal{E}'$ is $\mathcal{E}_{12}$ (type F_4). Let $A'=A_{40}$ (type E_8) and $A'=A_{12}$ (type F_4). Since A' is clean we have

$H_c^*(\gamma',\mathcal{E}'\otimes Y)=H_c^*(\bar\gamma',A'\otimes Y)=H_c^*(G,A'\otimes Y)$

and this is 0 by 1.3(a).

Using (b) and $H_c^*(\bar\gamma,A\otimes Y)=0$ we deduce that $H_c^*(\gamma,A\otimes Y)=0$ hence $H_c^*(\gamma,\mathcal{E}\otimes Y)=0$. Thus $H_c^{2\delta+i_0}(\gamma,\mathcal{E}\otimes Y)=0$. We have a spectral sequence with $E_2^{r,s}=H_c^r(\gamma,\mathcal{E}\otimes\mathcal{H}^sY)$ which converges to $H_c^{r+s}(\gamma,\mathcal{E}\otimes Y)$. We show that

$E_2^{r,s}=0$ if $s>i_0$.

It is enough to show that $H_c^*(\gamma,\mathcal{E}\otimes\mathcal{L})=0$ for any noncuspidal irreducible G-local system $\mathcal{L}$ on γ. This follows by applying the argument in line 8 and the ones following it in the proof of [L2, II, 7.8] to $\Sigma=\gamma$ (a distinguished unipotent class) and to $\mathcal{F}=\mathcal{E}\otimes\mathcal{L}$ (an irreducible G-local system on γ not isomorphic to $\bar{\mathbf{Q}}_l$).

We have also $E_2^{r,s}=0$ if $r>2\delta$. It follows that $E_2^{2\delta,i_0}=E_3^{2\delta,i_0}=\dots=E_\infty^{2\delta,i_0}$. But $E_\infty^{2\delta,i_0}$ is a subquotient of $H^{2\delta+i_0}(\gamma,\mathcal{E}\otimes Y)$ hence it is zero. It follows that $0=E_2^{2\delta,i_0}=H_c^{2\delta}(\gamma,\mathcal{E}\otimes\mathcal{H}^{i_0}Y)$. This contradicts $H_c^{2\delta}(\gamma,\mathcal{E}\otimes\mathcal{H}^{i_0}Y)\neq 0$. This proves (a).

Note that a property like (a) appeared (in good characteristic) in the work of Shoji [Sh1] and Beynon-Spaltenstein [BS].

2. Preliminaries to the proof

2.1. Let $\mathcal{B}$ be the variety of Borel subgroups of G. Let $\mathbf{W}$ be a set indexing the set of orbits of G acting on $\mathcal{B}\times\mathcal{B}$ by $g:(B,B')\mapsto(gBg^{-1},gB'g^{-1})$. For $w\in\mathbf{W}$ we write $\mathcal{O}_w$ for the corresponding G-orbit in $\mathcal{B}\times\mathcal{B}$. Define $\underline{l}:\mathbf{W}\to\mathbf{N}$ by $\underline{l}(w)=\dim\mathcal{O}_w-\dim\mathcal{B}$. Then $\mathbf{W}$ has a natural structure of (finite) Coxeter group with length function $\underline{l}$ (see for example [L3,0.2]); it is the Weyl group of G.

For $w\in\mathbf{W}$ let $\mathfrak{B}_w=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\mathcal{O}_w\}$. Define $\pi_w:\mathfrak{B}_w\to G$ by $\pi_w(g,B)=g$. Let $K_w=\pi_{w!}\bar{\mathbf{Q}}_l$, a complex of sheaves on G. Let

$\mathfrak{B}_{\le w}=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\cup_{y\le w}\mathcal{O}_y\}$,

$\mathfrak{B}_{<w}=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\cup_{y<w}\mathcal{O}_y\}$.

Define $\pi_{\le w}:\mathfrak{B}_{\le w}\to G$ by $\pi_{\le w}(g,B)=g$. Define $\pi_{<w}:\mathfrak{B}_{<w}\to G$ by $\pi_{<w}(g,B)=g$. Let $K_{\le w}=\pi_{\le w!}(IC(\mathfrak{B}_{\le w},\bar{\mathbf{Q}}_l))$, a complex of sheaves on G (here $\bar{\mathbf{Q}}_l$ is viewed as a local system on the open dense smooth subvariety $\mathfrak{B}_w$ of $\mathfrak{B}_{\le w}$). Let $K_{<w}=\pi_{<w!}(IC(\mathfrak{B}_{\le w},\bar{\mathbf{Q}}_l))$, a complex of sheaves on G.

2.2. We show:

(a) *Let $y \in \mathbf{W}$. We have ${}^pH^jK_y = 0$ if $j < \Delta + \underline{l}(y)$.*
We can assume that the result holds when G is replaced by a Levi subgroup of a proper parabolic subgroup of G. We can also assume that G is semisimple. We first prove (a) for y such that y has minimal length in its conjugacy class. If y is elliptic and it has minimal length in its conjugacy class in $\mathbf{W}$ then, according to [L5, 0.3(c)], π_y is affine and using [BBD, 4.1.1] we have ${}^pH^jK_y[\Delta + \underline{l}(y)]) = 0$ if $j < 0$ hence ${}^pH^{j+\Delta+\underline{l}(y)}K_y = 0$ if $j < 0$ so that (a) holds for y. If y is non-elliptic and it has minimal length in its conjugacy class in $\mathbf{W}$ then, according to [GP, 3.2.7], y is contained in the subgroup $\mathbf{W}'$ of $\mathbf{W}$ generated by a proper subset of the set of simple reflections of $\mathbf{W}$. Then $\mathbf{W}'$ can be viewed as the Weyl group of a Levi subgroup L of a proper parabolic subgroup P of G. Define $K_{y,L}$ in terms of y, L in the same way as K_y was defined in terms of y, G. For any j we have

(b) $\mathrm{ind}_P^G({}^pH^jK_{y,L}) = {}^pH^{j+\Delta-\Delta'}K_y$.

where ind_P^G is as in [L2, 4.1] and $\Delta' = \dim L$. (This is proved along the same lines as [L2, I, 4.8(a)].) If $j < \Delta + \underline{l}(y)$ we have $j' < \Delta' + \underline{l}$ where $j' = j - \Delta + \Delta'$ hence ${}^pH^{j'}K_{y,L} = 0$ so that $\mathrm{ind}_P^G({}^pH^{j'}K_{y,L}) = 0$ and

$0 = {}^pH^{j'+\Delta-\Delta'}K_y = {}^pH^jK_y$

so that (a) holds for y.

We now prove (a) for any $y \in \mathbf{W}$ by induction on $\underline{l}(y)$. If $\underline{l}(y) = 0$ then $y = 1$ has minimal length in its conjugacy class and (a) holds. Now assume that $\underline{l}(y) > 0$ and that the result is known for y' such that $\underline{l}(y') < \underline{l}(y)$. By [GP, 3.2.9] we can find a sequence $y = y_0, y_1, \dots, y_t$ in $\mathbf{W}$ such that $\underline{l}(y_0) \ge \underline{l}(y_1) \ge \dots \ge \underline{l}(y_t)$, y_t has minimal length in its conjugacy class and for any $i \in [0, t-1]$ we have $y_{i+1} = s_iy_is_i$ for some simple reflection s_i. Since (a) is already known for y_t it is enough to verify the following statement:

(c) *if $i \in [0, t-1]$ and (a) holds for $y = y_{i+1}$ then (a) holds for $y = y_i$.*

If $\underline{l}(y_i) = \underline{l}(y_{i+1})$ then, by an argument similar to that in [L3, 5.3], we see that there exists an isomorphism $\mathfrak{B}_{y_i} \xrightarrow{\sim} \mathfrak{B}_{y_{i+1}}$ commuting with the G-actions and commuting with $\pi_{y_i}, \pi_{y_{i+1}}$; hence $K_{y_i} = K_{y_{i+1}}$ and (c) follows in this case. Thus we can assume that $\underline{l}(y_i) > \underline{l}(y_{i+1})$ so that $\underline{l}(y_i) = \underline{l}(y_{i+1}) + 2$. We set $z = y_i, z' = y_{i+1}, s = s_i$. For $(g, B) \in \mathfrak{B}_z$ we can find uniquely B_1, B_2 in $\mathcal{B}$ such that $(B, B_1) \in \mathcal{O}_s$, $(B_1, B_2) \in \mathcal{O}_{z'}$, $(B_2, gBg^{-1}) \in \mathcal{O}_s$. Adapting an idea in [DL, §1], we define a partition $\mathfrak{B}_z = \mathfrak{B}_z^1 \cup \mathfrak{B}_z^2$ by

$$\mathfrak{B}_z^1 = \{(g, B) \in \mathfrak{B}_z; B_2 = gB_1g^{-1}\}, \mathfrak{B}_z^2 = \{(g, B) \in \mathfrak{B}_z; B_2 \ne gB_1g^{-1}\}.$$

Let $\pi_z^1 : \mathfrak{B}_z^1 \to G$, $\pi_z^2 : \mathfrak{B}_z^1 \to G$ be the restrictions of π_z. Let $K_z^1 = \pi_{z!}^1\bar{\mathbf{Q}}_l$, $K_z^2 = \pi_{z!}^2\bar{\mathbf{Q}}_l$. It is enough to show that ${}^pH^jK_z^1 = 0$ and ${}^pH^jK_z^2 = 0$ if $j < \Delta + \underline{l}(z)$. Now $(g, B) \mapsto (g, B_1)$ is a morphism $\mathfrak{B}_z^1 \to \mathfrak{B}_{z'}$, in fact an affine line bundle. It follows that $K_z^1 = K_{z'}[-2]$. Thus ${}^pH^jK_z^1 = {}^pH^{j-2}K_{z'}$. This is 0 for $j < \Delta + \underline{l}(z)$ since $j - 2 < \Delta + \underline{l}(z')$. Now $(g, B) \mapsto (g, B_2)$ is a morphism $\mathfrak{B}_z^2 \to \mathfrak{B}_{sz'}$, in fact a line bundle with the zero-section removed. It follows that for any j we have an exact sequence of perverse sheaves on G:

$$ {}^pH^{j-1}K_{sz} \to {}^pH^jK_z^2 \to {}^pH^j(K_{sz}[-2]).$$

Since $\underline{l}(sz') = \underline{l}(z) - 1$ we know that (a) holds for sz. If $j < \Delta + \underline{l}(z)$ then $j-1 < \Delta+\underline{l}(sz')$ hence ${}^pH^{j-1}K_{sz'} = 0$ and ${}^pH^j(K_{sz'}[-2]) = {}^pH^{j-2}K_{sz'} = 0$; the exact sequence above then shows that ${}^pH^jK_z^2 = 0$. This completes the inductive proof of (c) hence that of (a). (A somewhat similar strategy was employed in [OR] to prove a vanishing property for the cohomology of the varieties X_w of [DL]; I thank X.He for pointing out the reference [OR] to me.)

2.3. We show:

(a) *Let $y \in \mathbf{W}$ and let A be a character sheaf on G such that $(A : \oplus_j {}^pH^jK_{y'}) = 0$ for any $y' \in \mathbf{W}$, $y' < y$. Then $(A : {}^pH^jK_y) = 0$ for any $j \ne \Delta + \underline{l}(y)$. Moreover, if $j = \Delta + \underline{l}(y)$, there exists a (necessarily unique) subobject ${}^pH^jK_y^A$ of ${}^pH^jK_y$ such that ${}^pH^jK_y/{}^pH^jK_y^A$ is semisimple, A-isotypic and $(A : {}^pH^jK_y^A) = 0$.*

From our assumption we deduce (as in [L2, III, 12.7]) that $(A : \oplus_j {}^pH^jK_{<y}) = 0$. Hence the obvious morphism $\phi_j : {}^pH^jK_y \to {}^pH^jK_{\le y}$ satisfies $(A : \ker\phi_j) = 0$, $(A : \mathrm{coker}\phi_j) = 0$. In particular, $(A : {}^pH^jK_{\le y}) = (A : {}^pH^jK_y)$ for any j. Since $\pi_{\le y}$ is proper, ${}^pH^jK_{\le y}$ is semisimple, see [BBD]. Hence there is a unique direct sum decomposition of perverse sheaves ${}^pH^jK_{\le y} = {}^pH^jK_{\le y,A} \oplus M$ such that ${}^pH^jK_{\le y,A}$ is semisimple, A-isotypic and $(A : M) = 0$. Let

$u : {}^pH^jK_{\le y,A} \oplus M \to {}^pH^jK_{\le y,A}$

be the first projection. The composition

${}^pH^jK_y \xrightarrow{\phi_j} {}^pH^jK_{\le y,A} \oplus M \xrightarrow{u} {}^pH^jK_{\le y,A}$

is surjective (the image of ϕ_j contains ${}^pH^jK_{\le y,A}$ since $(A : \mathrm{coker}\phi_j) = 0$). Let ${}^pH^jK_y^A$ be the kernel of this composition. Then ${}^pH^jK_y/{}^pH^jK_y^A \cong {}^pH^jK_{\le y,A}$ hence ${}^pH^jK_y/{}^pH^jK_y^A$ is semisimple, A-isotypic. Moreover

$$\begin{aligned}(A : {}^pH^jK_y^A) &= (A : {}^pH^jK_y) - (A : {}^pH^jK_y^A {}^pH^jK_y)\\ &= (A : {}^pH^jK_{\le y}) - (A : {}^pH^jK_{\le y,A}) = (A : M) = 0.\end{aligned}$$

By the Lefschetz hard theorem [BBD, 5.4.10] we have for any j':

${}^pH^{-j'}(K_{\le y}[\Delta + \underline{l}(y)]) \cong {}^pH^{j'}(K_{\le y}[\Delta + \underline{l}(y)])$

hence for any j, ${}^pH^jK_{\le y} \cong {}^pH^{2\Delta+2\underline{l}(y)-j}K_{\le y}$. It follows that

${}^pH^jK_{\le y,A} \cong {}^pH^{2\Delta+2\underline{l}(y)-j}K_{\le y,A}$

so that

(b) ${}^pH^jK_y/{}^pH^jK_y^A \cong {}^pH^{2\Delta+2\underline{l}(y)-j}K_y/{}^pH^{2\Delta+2\underline{l}(y)-j}K_y^A$.

Using 2.2(a) we have ${}^pH^jK_y = 0$ if $j < \Delta + \underline{l}(y)$. Hence ${}^pH^jK_y/{}^pH^jK_y^A = 0$ if $j < \Delta + \underline{l}(y)$. Using (b) we deduce ${}^pH^jK_y/{}^pH^jK_y^A = 0$ if $j > \Delta + \underline{l}(y)$. Thus ${}^pH^jK_y/{}^pH^jK_y^A = 0$ if $j \ne \Delta + \underline{l}(y)$. Since $(A : {}^pH^jK_y^A) = 0$ it follows that $(A : {}^pH^jK_y) = 0$ if $j \ne \Delta + \underline{l}(y)$. This completes the proof of (a).

2.4. In this subsection we assume that G is adjoint. Let w be an elliptic element of $\mathbf{W}$ which has minimal length in its conjugacy class C. We assume that the unipotent class $\gamma = \Phi(C)$ in G (Φ as in [L3, 4.1]) is distinguished and that $\det(1-$

w) is a power of p (the determinant is taken in the reflection representation of $\mathbf{W}$). According to [L4, 0.2],

(a) *the variety $\pi_w^{-1}(\gamma)$ is a single G-orbit for the G-action $x : (g, B) \mapsto (xgx^{-1}, xBx^{-1})$ on $\mathfrak{B}_w$.*

We show:

(b) *$K_w[2\underline{l}(w)]|_\gamma \cong \oplus_{\mathcal{E}} \mathcal{E}^{\oplus \mathrm{rk}(\mathcal{E})}$ where $\mathcal{E}$ runs over all irreducible G-local systems on γ (up to isomorphism).*

Let $(g, B) \in \pi_w^{-1}(\gamma)$ and let $Z_G(g)$ be the centralizer of g. According to [L3, 4.4(b)] we have

(c) $\dim Z_G(g) = \underline{l}(w)$.

We have a commutative diagram

$$\begin{array}{ccc} G & \xrightarrow{\beta} & G/Z_G(g) \\ {\scriptstyle\alpha}\downarrow & & {\scriptstyle\alpha'}\downarrow \\ \pi_w^{-1}(\gamma) & \xrightarrow{\sigma} & \gamma \end{array}$$

where β is the obvious map, $\alpha(x) = (xgx^{-1}, xBx^{-1})$, $\alpha'(x) = xgx^{-1}$, $\sigma(g', B') = g'$. Now α is surjective by (a); it is also injective since by [L3, 5.2] the isotropy groups of the G-action on $\mathfrak{B}_w$ are trivial (we use our assumption on $\det(1-w)$). Thus α is a bijective morphism so that $\alpha_!\bar{\mathbf{Q}}_l = \bar{\mathbf{Q}}_l$. Hence

$$K_w[2\underline{l}(w)]|_\gamma = \sigma_!\bar{\mathbf{Q}}_l[2\underline{l}(w)] = \sigma_!\alpha_!\bar{\mathbf{Q}}_l[2\underline{l}(w)] = \alpha'_!\beta_!\bar{\mathbf{Q}}_l[2\underline{l}(w)].$$

We now factorize β as follows:

$G \xrightarrow{\beta_1} G/Z_G(g)^0 \xrightarrow{\beta_2} G/Z_G(g)$.

Since all fibres of β_1 are isomorphic to $Z_G(g)^0$ (an affine space of dimension $\underline{l}(w)$, see (c)), we have $\beta_!\bar{\mathbf{Q}}_l \cong \bar{\mathbf{Q}}_l[-2\underline{l}(w)]$. Thus

$$K_w[2\underline{l}(w)]|_\gamma \cong \alpha'_!\beta_{2!}\bar{\mathbf{Q}}_l = (\alpha'\beta_2)_!\bar{\mathbf{Q}}_l.$$

Now $\alpha'\beta_2$ is a principal covering with (finite) group $Z_G(g)/Z_G(g)^0$; (b) follows. (The proof above has some resemblance to the proof of [L2, IV, 21.11].)

3. Completion of the proof

3.1. In this section (except in 3.10) we assume that $p = 2$ and that G is of type E_8 or F_4. We have the following result:

(a) *Let $y \in \mathbf{W}$ be an elliptic element of minimal length in its conjugacy class and let $i \in I$ be such that $\pi_y^{-1}(\gamma_i) \neq \emptyset$. Then $\underline{l}(y) \geq i$.*

Indeed, from [L3, 5.7(iii)] we have $\dim \gamma_i \geq \Delta - \underline{l}(y)$ and it remains to use that $\dim \gamma_i = \Delta - i$.

3.2. Let $i \ne i'$ in I. Then

(a) $\oplus_j \mathcal{H}^j A_{i'}|_{\gamma_i}$ *does not contain* $\mathcal{E}_i$ *as a direct summand except possibly when* $i = 40, i' = 20$ *(type* E_8*) and* $i = 12, i' = 8$ *(type* F_4*).*

If $i' \ne 20$ (type E_8) and $i' \ne 8$ (type F_4) this follows from the cleanness of $A_{i'}$. If $i' = 20, i \ne 40$ (type E_8) and $i' = 8, i \ne 12$ (type F_4) this follows from the fact that $\gamma_i \not\subset \bar{\gamma}_{i'}$ except when $i' = 20, i = 22$ (type E_8) when the result follows from 1.4(a).

Note that if $i' = 10$ (type E_8) and $i' = 4$ (type F_4) then

(b) $\oplus_j \mathcal{H}^j A'_{i'}|_{\gamma_i} = 0$

by the cleanness of $A'_{i'}$. If $i = 10$ (type E_8) and $i = 4$ (type F_4) then

(c) $\oplus_j \mathcal{H}^j A_{i'}|_{\gamma_i} = 0$

since $\gamma_i \not\subset \bar{\gamma}_{i'}$.

3.3. We show:

(a) *Assume that* $i \in I$, $y \in \mathbf{W}$, $\underline{l}(y) < i$. *Assume also that* $i \ne 40$ *(type* E_8*) and* $i \ne 12$ *(type* F_4*). Then* $(A_i : \oplus_j {}^p H^j K_y) = 0$. *If, in addition,* $i = 10$ *for type* E_8 *and* $i = 4$ *for type* F_4 *then* $(A'_i : \oplus_j {}^p H^j K_y) = 0$.

Assume that the first assertion of (a) is false. Then we can find $y' \in \mathbf{W}$ such that $\underline{l}(y') < i$, $(A_i : \oplus_j {}^p H^j K_{y'}) \ne 0$ and $(A_i : \oplus_j {}^p H^j K_{y''}) = 0$ for any $y'' \in \mathbf{W}$ with $y'' < y'$. Using 2.3(a) we see that $(A_i : {}^p H^j K_{y'}) = 0$ for any $j \ne \Delta + \underline{l}(y')$ hence $(A_i : {}^p H^j K_{y'}) \ne 0$ for $j = \Delta + \underline{l}(y')$. It follows that $\sum_j (-1)^j (A_i : {}^p H^j K_{y'}) \ne 0$. Using [L2, I, 6.5] we deduce that $\sum_j (-1)^j (A_i : {}^p H^j K_{y'_1}) \ne 0$ for any $y'_1 \in \mathbf{W}$ that is conjugate to y'. If y' is not elliptic then some y'_1 in the conjugacy class of y' is contained in the subgroup $\mathbf{W}'$ of $\mathbf{W}$ generated by a proper subset of the set of simple reflections of $\mathbf{W}$. Then $\mathbf{W}'$ can be viewed as the Weyl group of a Levi subgroup L of a proper parabolic subgroup P of G. Define $K_{y'_1,L}$ in terms of y'_1, L in the same way as K_y was defined in terms of y, G. We have $(A_i : \oplus_j {}^p H^j K_{y'_1}) \ne 0$. From this and from the equality 2.2(b) (for y'_1 instead of y) we deduce that $(A_i : \mathrm{ind}_P^G({}^p H^j K_{y'_1,L})) \ne 0$ for some j. Hence $(A_i : \mathrm{ind}_P^G(\tilde{A})) \ne 0$ for some character sheaf $\tilde{A}$ on L; this contradicts the fact that A_i is a cuspidal character sheaf. We see that y' is elliptic. If the conjugacy class of y' contains an element y'_2 such that $\underline{l}(y'_2) < \underline{l}(y')$ then using again [L2, I, 6.5], we deduce from $\sum_j (-1)^j (A_i : {}^p H^j K_{y'}) \ne 0$ that $\sum_j (-1)^j (A_i : {}^p H^j K_{y'_2}) \ne 0$ hence $(A_i : {}^p H^j K_{y'_2}) \ne 0$, contradicting the choice of y'. We see that y' has minimal length in its conjugacy class.

For any G-equivariant perverse sheaf M on G we set $\chi_i(M) = \sum_j (-1)^j (\mathcal{E}_i : \mathcal{H}^j M|_{\gamma_i})$ where $(\mathcal{E}_i :?)$ denotes multiplicity in a G-local system. For any noncuspidal character sheaf X on G we have $\chi_i(X) = 0$, see 1.6(a). For any cuspidal character sheaf X on G with nonunipotent support we have clearly $\chi_i(X) = 0$.

If $i' \in I - \{i\}$ then $\chi_i(A_{i'}) = 0$ by 3.2. Also, if $i' = 10$ (type E_8) and $i' = 4$ (type F_4) and $i' \ne i$ then $\chi_i(A'_{i'}) = 0$ by 3.2.

From the definition we have $\chi_i(A_i) \ne 0$. Since $(A_i : {}^p H^j K_{y'}) = 0$ for any $j \ne \Delta + \underline{l}(y')$ and $(A_i : {}^p H^j K_{y'}) \ne 0$ for $j = \Delta + \underline{l}(y')$ it follows that $\chi_i({}^p H^j K_{y'}) \ne 0$ for

$j\ne\Delta+\underline{l}(y')$ and $\chi_i({}^pH^jK_{y'})=0$ for $j=\Delta+\underline{l}(y')$. Hence $\sum_j(-1)^j\chi_i({}^pH^jK_{y'})\ne 0$. Hence $\sum_j(-1)^j(\mathcal{E}_i:\mathcal{H}^jK_{y'}|_{\gamma_i})\ne 0$. It follows that $K_{y'}|_{\gamma_i}\ne 0$ so that $\pi_{y'}^{-1}(\gamma_i)\ne\emptyset$. Using 3.1(a) we deduce that $\underline{l}(y')\ge i$. This contradicts $\underline{l}(y')<i$ and proves the first assertion of (a). The proof of the second assertion of (a) is entirely similar,

3.4. We now prove a weaker version of 3.3(a) assuming that $i=40$ (type E_8) and $i=12$ (type F_4).

(a) *If $y\in\mathbf{W}$, $\underline{l}(y)<20$ (type E_8) and $\underline{l}(y)<8$ (type F_4) then $(A_i:\oplus_j{}^pH^jK_y)=0$.*

We go through the proof of 3.3(a). The first two paragraphs remain unchanged. In the third pragraph, the sentence

"If $i'\in I-\{i\}$ then $\chi_i(A_{i'})=0$ by 3.2."

must be modified as follows:

"If $i'\in I-\{i\}$ and $i'\ne 20$ (type E_8) and $i'\ne 8$ (type F_4) then $\chi_i(A_{i'})=0$ by 3.2. Moreover, if $i'=20$ (type E_8) and $i'=8$ (type F_4) then by 3.3(a), $(A_{i'}:{}^pH^jK_{y'})=0$ for any j, since $\underline{l}(y')<20$ (type E_8) and $\underline{l}(y')<8$ (type F_4)". Then the fourth paragraph remains unchanged and (a) is proved.

3.5. For any $i\in I$ we consider the conjugacy class C_i of $\mathbf{W}$ whose elements have the following characteristic polynomial in the reflection representation $\mathcal{R}$ of $\mathbf{W}$:

(type E_8): q^8-q^4+1 (if $i=10$), $(q^4-q^2+1)^2$ (if $i=20$), $(q^2-q+1)^2(q^4-q^2+1)$ (if $i=22$), $(q^2-q+1)^4$ (if $i=40$);

(type F_4): (q^4-q^2+1) (if $i=4$), q^4+1 (if $i=6$), $(q^2-q+1)^2$ (if $i=8$), $(q^2+1)^2$ (if $i=12$).

We choose an element w_i of minimal length in C_i. Then $\underline{l}(w_i)=i$. Note that w_i is elliptic and $\det(1-w_i,\mathcal{R})$ is 1 (type E_8) and a power of 2 (type F_4).

Let Φ be the (injective) map from the set of elliptic conjugacy classes in $\mathbf{W}$ to the set of unipotent classes in G defined in [L3, 4.1]. We have $\Phi(C_i)=\gamma_i$.

Note that the correspondence between C_i and (the characteristic zero analogue of) γ_i appeared in another context in the (partly conjectural) tables of Spaltenstein [Sp2].

3.6. In this subsection we set $i=20,i'=40$ (type E_8) and $i=8,i'=12$ (type F_4). Let $w=w_i$. We have the following results.

(a) *If $j\ne\Delta+i$ then $(A_i:{}^pH^jK_w)=0$ and $(A_{i'}:{}^pH^jK_w)=0$.*

(b) *If $j=\Delta+i$ then $(A_i:{}^pH^jK_w)=1$; there exists a unique subobject Z of ${}^pH^jK_w$ such that $(A_i:Z)=0$ and ${}^pH^jK_w/Z\cong A_i$ and there exists a unique subobject Z' of ${}^pH^jK_w$ such that $(A_{i'}:Z')=0$ and ${}^pH^jK_w/Z'$ is semisimple, $A_{i'}$-isotypic.*

(a) follows from 2.3(a) applied with $y=w$ and with A equal to A_i or $A_{i'}$. (The assumptions of 2.3(a) are satisfied by 3.3(a), 3.4(a).) As in the proof of 3.3(a) we see that for any character sheaf A' not isomorphic to A_i we have $\chi_i(A')=0$. From 2.4(b) we see that $\sum_j(-1)^j\chi_i({}^pH^{j+2i}K_w)=1$ (we use that $\mathcal{E}_i$ has rank 1). Hence $\sum_j(-1)^j(A_i:{}^pH^jK_w)\chi_i(A_i)=1$ that is $(-1)^{\Delta+i}(A_i:{}^pH^{\Delta+i}K_w)\chi_i(A_i)=1$.

Since $\chi_i(A_i) = \pm 1$ it follows that $(A_i : {}^pH^{\Delta+i}K_w) = 1$ proving the first assertion of (b). The remaining assertions of (b) follow from 2.3(a) applied with $y = w$ and with A equal to A_i or $A_{i'}$.

3.7. In the setup of 3.6 we show:

(a) *for any j, $\oplus_k \mathcal{H}^k({}^pH^jK_w)|_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand.*

Assume first that $j \neq \Delta + i$. It is enough to show that for any character sheaf X such that $(X : {}^pH^jK_w) \neq 0$, $\oplus_k \mathcal{H}^k X_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. If X is noncuspidal this follows from 1.6(a); if X is cuspidal, it must be different from A_i or $A_{i'}$ (see 3.6(a)) and the result follows from the cleanness of cuspidal character sheaves other than A_i.

Assume now that for some k, $\mathcal{H}^k({}^pH^{\Delta+i}K_w)|_{\gamma_{i'}}$ contains $\mathcal{E}_{i'}$ as a direct summand. This, and the previous paragraph, imply that for some k, $\mathcal{H}^k(K_w)|_{\gamma_{i'}}$ contains $\mathcal{E}_{i'}$ as a direct summand. In particular $K_w|_{\gamma_{i'}} \neq 0$ so that $\pi_w^{-1}(\gamma_{i'}) \neq \emptyset$. Using 3.1(a) we deduce that $\underline{l}(w) \geq i'$ that is, $i \geq i'$. This contradiction proves (a).

3.8. We preserve the setup of 3.6. We have ${}^pH^{\Delta+i}K_w = Z + Z'$ since

${}^pH^{\Delta+i}K_w/(Z + Z')$

is both A_i-isotypic and $A_{i'}$-isotypic. (It is a quotient of ${}^pH^{\Delta+i}K_w/Z$ which is A_i-isotypic and a quotient of ${}^pH^{\Delta+i}K_w/Z'$ which is $A_{i'}$-isotypic.) As in the proof of 3.7(a) we see that all composition factors X of $Z \cap Z'$ (which are necessarily not isomorphic to A_i or $A_{i'}$) satisfy the condition that $\oplus_k \mathcal{H}^k(X)|_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. It follows that $\oplus_k \mathcal{H}^k(Z \cap Z')_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. Using this and 3.7(a) we deduce that $\oplus_k \mathcal{H}^k({}^pH^{\Delta+i}K_w/(Z \cap Z'))_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. Since ${}^pH^{\Delta+i}K_w = Z + Z'$, the natural map ${}^pH^{\Delta+i}K_w/(Z \cap Z') \to ({}^pH^{\Delta+i}K_w/Z) \oplus ({}^pH^{\Delta+i}K_w/Z')$ is an isomorphism. It follows that

(a) $\oplus_k \mathcal{H}^k({}^pH^{\Delta+i}K_w/Z)_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand;

(b) $\oplus_k \mathcal{H}^k({}^pH^{\Delta+i}K_w/Z')_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand.

Since ${}^pH^{\Delta+i}K_w/Z \cong A_i$, we see that (a) (together with 1.5(a)) proves the cleanness of A_i thus completing the proof of Theorem 1.2. Since ${}^pH^{\Delta+i}K_w/Z'$ is a direct sum of copies of $A_{i'}$ and $\oplus_k \mathcal{H}^k(A_{i'})_{\gamma_{i'}} = \mathcal{E}_{i'}$ we see that (b) implies ${}^pH^{\Delta+i}K_w/Z' = 0$. This, together with 3.6(a),(b) implies that

(c) $(A_{i'} : {}^pH^jK_w) = 0$ *for any j.*

3.9. In view of the cleanness of G, we can restate 3.2(a) in a stronger form:

(a) *Let $i \neq i'$ in I. Then $\oplus_j \mathcal{H}^j A_{i'}|_{\gamma_i}$ does not contain $\mathcal{E}_i$ as a direct summand.*

Using this the proof of 3.3(a) applies in greater generality and yields the following result.

(b) *Assume that $i \in I$, $y \in \mathbf{W}$, $\underline{l}(y) < i$. Then $(A_i : \oplus_j {}^pH^jK_y) = 0$. If, in addition, $i = 10$ for type E_8 and $i = 4$ for type F_4 then $(A_i' : \oplus_j {}^pH^jK_y) = 0$.*

From (b), 2.3(a) and 2.4(b) we deduce as in 3.6 the following result for any $i \in I$:

(c) *If $j \neq \Delta + i$ then $(A_i : {}^pH^jK_{w_i}) = 0$; if $j = \Delta + i$ then $(A_i : {}^pH^jK_{w_i}) = 1$ and there exists a unique subobject Z of ${}^pH^jK_{w_i}$ such that $(A_i : Z) = 0$ and*

${}^pH^jK_{w_i}/Z \cong A_i$.
The same result holds for $i = 10$ (type E_8) and $i = 4$ (type F_4) if A_i is replaced by A'_i.

3.10. Note that, once Theorem 1.2 is known, the parity property [L2, III, (15.13.1)] can be established for a reductive group in any characteristic as in [L2]. (Incidentally, note that 3.9(c) establishes the parity property for the character sheaves A_i for $p = 2$, type E_8 or F_4.) Using this we see that essentially the same proof as in [L2] establishes [L2, V, Theorems 23.1, 24.4, 25.2, 25.6] (but not [L2, V, Theorem 24.8]) for a reductive group in any characteristic.

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Department of Mathematics, M.I.T., Cambridge, MA 02139